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COMMENT ON: IDENTIFICATION IN THE LINEAR ERRORS IN VARIABLES MODEL

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1. Introduction

Kapteyn and Wansbeek [1] considered the following multiple linear regression model with errors in variables:

(1.1) $y_j = \xi_j'\beta + \varepsilon_j,$ $(j = 1,\ldots,n)$

(1.2) $x_j = \xi_j + \upsilon_j,$

where ξ_j, x_j, υ_j and β are k-vectors, y_j, ε_j are scalars. The ξ_j are unobservable variables: instead the x_j are observed. The measurement errors υ_j are unobservable as well and it is assumed that $\upsilon_j \sim N(0,\Omega)$ and $\varepsilon_j \sim N(0,\sigma^2)$ for all j. The υ_j and ε_j are mutually independent and independent of ξ_j. The ξ_j are considered as random drawings from some, as yet unspecified, multivariate distribution with zero mean.
For the case k=1 Reiersøl [2] has shown that normality of ξ_j is the only distributional assumption which spoils identification. For the case k > 1 and the components of ξ_j are mutually independent, Wilassen [3] has shown that none of the components of ξ_j should be normally distributed to guarantee identifiability of β. Kapteyn and Wansbeek [1] did not assume independency of the components of ξ_j and they stated the following proposition: the parameter vector β is identified if and only if there does not exist a lineair combination of ξ_j which is normally distributed. The necessity part in this proposition is incorrect, i.e. it may well be that a normally distributed linear combination of ξ_j does not spoil the identifiability of β. Here I present necessary and sufficient conditions for identification of β.

2. Statement of the Result and Proof

Proposition: Under the assumptions above, the parameter vector β is identified if and only if there does not exist a nonsingular matrix $A = (a_1, A_2)$ such that $\xi_j' a_1$ is distributed normally and independently of $\xi_j' A_2$.

Proof: We first show that nonidentifiability of β implies the existence of the matrix A. Let s be a scalar and t a k-vector. The characteristic function $\phi_{y_j,x_j}(s,t)$ of y_j and x_j is

$$(2.1) \qquad \phi_{y_j,x_j}(s,t) = \exp\{-\tfrac{1}{2}(\sigma^2 s^2 + t'\Omega t)\}\ \phi_\xi(\beta s + t) \ ,$$

where $\phi_\xi(.)$ is the characteristic function of ξ_j. Assuming that β is not fully identified amounts to saying that there exist parameter sets $\{\beta, \sigma^2, \Omega\}$ and $\{\beta^*, \sigma^{*2}, \Omega^*\}$, with $\beta \neq \beta^*$, generating the same distribution of y_j, x_j. Consequently $\phi_{y_j,x_j}(s,t)$ should be the same for both sets of parameters:

$$(2.2) \qquad \exp\{-\tfrac{1}{2}(\sigma^2 s^2 + t'\Omega t)\}\ \phi_\xi(\beta s + t) = \exp\{-\tfrac{1}{2}(\sigma^{*2} s^2 + t'\Omega^* t)\}\ \phi_\xi^*(\beta^* s + t).$$

Let $\ell = \beta^* s + t$, then $\phi_\xi(\beta s + t) = \phi_\xi((\beta - \beta^*)s + \ell) = \phi_{\xi'(\beta-\beta^*),\xi}(s,\ell)$. Thus (2.2) caries over into

$$(2.3) \qquad \phi_{\xi'(\beta-\beta^*),\xi}(s,\ell) = \exp\{-\tfrac{1}{2}[s^2(\sigma^{*2} - \sigma^2) +$$

$$+ (\ell - \beta^* s)'(\Omega^* - \Omega)(\ell - \beta^* s)]\}\ \phi_\xi^*(\ell).$$

The characteristic function corresponding to the marginal distribution of $\xi_j'(\beta - \beta^*)$ is found by setting $\ell = 0$

$$(2.4)\qquad \phi_{\xi'(\beta-\beta^*)}(s) = \exp\{-\tfrac{1}{2}s^2(\sigma^{*2}-\sigma^2+\beta^{*\prime}(\Omega^*-\Omega)\beta^*)\},$$

which is the characteristic function of a normally distributed variable.

In addition to this result, which was obtained by Kapteyn and Wansbeek [1], it will now be shown that nonidentifiability of β also implies the existence of a matrix A_2 such that (a_1,A_2) is nonsingular and $\xi_j'a_1$ is distributed independently of $\xi_j'A_2$. The characteristic function corresponding to the marginal distributions of ξ_j is found by setting s=0 in (2.3):

$$(2.5)\qquad \phi_{\xi}(\ell) = \exp\{-\tfrac{1}{2}\ell'(\Omega^*-\Omega)\ell\}\ \phi_{\xi}^*(\ell).$$

Thus, we may rewrite (2.3) as

$$(2.6)\qquad \phi_{\xi'(\beta-\beta^*),\xi}(s,\ell) = \phi_{\xi'(\beta-\beta^*)}(s)\ \phi_{\xi}(\ell)\ \exp\{s\beta^{*\prime}(\Omega^*-\Omega)\ell\}.$$

Let B be a $(k\times(k-1))$-matrix of full column rank such that $\beta^{*\prime}(\Omega^*-\Omega)B = 0$. Equality (2.6) holds for all possible values of s and ℓ. In particular (2.6) holds if we let ℓ vary such that $\ell = Bm$, where m is a (k-1)-vector:

$$(2.7)\qquad \phi_{\xi'(\beta-\beta^*),\xi}(s,Bm) = \phi_{\xi'(\beta-\beta^*)}(s)\ \phi_{\xi}(Bm),$$

or equivalently,

$$(2.8)\qquad \phi_{\xi'(\beta-\beta^*,B)}(s,m) = \phi_{\xi'(\beta-\beta^*)}(s)\ \phi_{\xi'B}(m).$$

Thus nonidentifiability of β implies the existence of a matrix $(\beta-\beta^*,B)$ such that $\xi_j'(\beta-\beta^*)$ is distributed normally and independently of $\xi_j'B$. If rank $(\beta-\beta^*,B) = k$ then a matrix A is given by $(\beta-\beta^*,B)$. In the trivial case where Rank $(\beta-\beta^*,B) = k-1$, the variable $\xi_j'(\beta-\beta^*)$ is distributed independently of itself and must therefore be equal to zero identically (which is also considered as a normal distribution). In that case any nonsingular matrix A whose first column equals $\beta-\beta^*$ will do.

To prove the necessity part of the Proposition we assume that there exists a nonsingular matrix $A = (a_1, A_2)$ such that $\xi_j' a_1$ is distributed normally and independently of $\xi_j' A_2$. If we substitute $t = A\ell = a_1\ell_1 + A_2\ell_2$ and $\beta = A\tilde{\beta} = a_1\tilde{\beta}_1 + A_2\tilde{\beta}_2$ (ℓ_1 and $\tilde{\beta}_1$ are scalars, ℓ_2 and $\tilde{\beta}_2$ are (k-1)-vectors) in (2.1), then the characteristic function of y_j, x_j takes the following form:

$$(2.9) \qquad \phi_{y_j, x_j}(s, A\ell) = \exp\{-\tfrac{1}{2}(\sigma^2 s^2 + \ell' A' \Omega A \ell)\}\ \phi_\xi(A(\tilde{\beta}s + \ell)).$$

The characteristic function $\phi_\xi(A(\tilde{\beta}s+\ell))$ can be rewritten as follows:

$$(2.10) \qquad \phi_\xi(A(\tilde{\beta}s+\ell)) = \phi_{\xi'A}(\tilde{\beta}s+\ell) = \phi_{\xi'a_1}(\tilde{\beta}_1 s+\ell_1)\ \phi_{\xi'A_2}(\tilde{\beta}_2 s+\ell_2)$$

$$= \exp\{-\tfrac{1}{2}(\tilde{\beta}_1 s+\ell_1)^2 \mathrm{Var}(\xi' a_1)\}\ \phi_{\xi'A_2}(\tilde{\beta}_2 s+\ell_2).$$

Using (2.10), (2.9) carries over into

$$(2.11) \qquad \phi_{y_j, x_j}(s_1 A\ell) = \exp\{-\tfrac{1}{2}(s,\ell')C(s,\ell')'\}\ \phi_{\xi'A_2}(\tilde{\beta}_2 s+\ell_2)\ ,$$

where

$$(2.12) \qquad C \equiv \begin{bmatrix} \sigma^2 & 0 \\ 0 & A'\Omega A \end{bmatrix} + \mathrm{Var}(\xi' a_1) \begin{bmatrix} \tilde{\beta}_1^2 & e_1'\tilde{\beta}_1 \\ e_1\tilde{\beta}_1 & e_1 e_1' \end{bmatrix} \quad .$$

The $\frac{1}{2}k(k+1) + 2$ nonzero elements of C are functions of $\frac{1}{2}k(k+1) + 3$ parameters in σ^2, Ω, $\tilde{\beta}_1$ and $\mathrm{Var}(\xi' a_1)$, whereas the function $\phi_{\xi'A_2}(\tilde{\beta}_2 s+\ell_2)$ is not affected by these parameters. Clearly, different parameter values generate the same distribution of y_j, x_j. The existence of a nonsingular transformation such that $\xi_j' a_1$ is distributed normally and independently of $\xi_j' A_2$ thus implies nonidentifiability of β. Q.E.D.

3. Discussion

Compared to Kapteyn and Wansbeek's proposition, the sufficiency part of the proposition proved here is stronger. Nonidentifiability does not only imply the existence of a normally distributed linear combination $\xi_j'a_1$, but also the existence of A_2 such that $\xi_j'a_1$ and $\xi_j'A_2$ are mutually independent. Consequently, the necessity part of their proposition must be wrong, because they fail to invoke the existence of a matrix A_2 such that $\xi_j'a_1$ and $\xi_j'A_2$ are mutually independent.

As an example, consider the model with two regressors ξ_{j1} and ξ_{j2}, the first of which is normally distributed, $\xi_{j1} \sim N(0,\sigma^2)$, and the second is a function of the first $\xi_{j2} = \xi_{j1}^2 - \sigma^2$. According to Kapteyn and Wansbeek this model would not be identified since ξ_{j1} is normally distributed. However, this point of view would be too pessimistic. Clearly there is no nonsingular transformation (a_1,a_2) such that $(\xi_{j1},\xi_{j2})a_1$ is distributed independently of $(\xi_{j1},\xi_{j2})a_2$ and so the model is identified.

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